

文章编号:1673-2049(2017)05-0196-07

考虑轴向位移的压弯 II 形梁剪力滞分析

周世军^{1,2}, 宋 刚¹

(1. 重庆大学 土木工程学院, 重庆 400045;

2. 重庆大学 山地城镇建设与新技术教育部重点实验室, 重庆 400045)

摘要:在截面纵向位移函数中引入截面轴向位移,以描述压弯作用下 II 形梁截面的变形状态。基于能量变分法导出轴向位移、竖向位移和剪力滞位移之间相互耦合的控制微分方程组,求得压弯作用下 II 形梁的位移解及其相应的边界条件。结合 ANSYS 软件,利用实体单元和导出的位移解及其边界条件分别对简支梁和悬臂梁进行分析,验证其有效性和可靠性。结果表明:剪力滞效应使得 II 形梁中性轴和形心轴相互分离,截面不再绕形心轴转动;在弯曲作用下,剪力滞使简支梁截面弯曲刚度减小,悬臂梁固定端一侧 1/4 跨度内截面弯曲刚度减小,悬臂梁自由端一侧 3/4 跨度内截面弯曲刚度增加;在压弯作用下,轴向压力引起的剪力滞使简支梁梁端附近截面弯曲刚度减小,悬臂梁自由端一侧截面弯曲刚度增加。

关键词:II 形梁;剪力滞;轴向位移;实体单元;弯曲刚度

中图分类号:TU 311

文献标志码:A

Analysis of Shear Lag of Compression-bending II-shaped Girder Considering Axial Displacement

ZHOU Shi-jun^{1,2}, SONG Gang¹

(1. School of Civil Engineering, Chongqing University, Chongqing 400045, China;

2. Key Laboratory of New Technology for Construction of Cities in Mountain Area, Chongqing University, Chongqing 400045, China)

Abstract: In the longitudinal displacement function of cross section, the axial displacement of cross section was introduced to describe the section deformation state of II shaped girder under axial compression and bending. Based on energy variation method, the governing differential equations coupled with axial displacement, vertical displacement and shear lag displacement were deduced, and the displacement solution of II shaped girder under axial compression and bending and its corresponding boundary conditions were obtained. Combined with ANSYS software, the simply supported girder and cantilever girder were analyzed by using solid element, the derived displacement solutions and its boundary conditions, and the validity and reliability of the method were verified. The results show that the shear lag effect makes the separation between neutral axis and centroidal axis on the section of II shaped girder, and the section no longer rotates around the centroidal axis. Under the action of bending, the shear lag decreases the flexural stiffness of the section of simply supported girder and the section within quarter span on the cantilever fixed side, and increases the flexural stiffness of the section within three quarter span

收稿日期:2017-07-20

基金项目:重庆市自然科学基金项目(CSTC,2010BB6048)

作者简介:周世军(1961-),男,甘肃天水人,教授,博士研究生导师,工学博士,E-mail:sjzhou8@163.com。

on the cantilever free side. Under the action of axial compression and bending, the shear lag caused by axial compression decreases the flexural stiffness of the section near the end of simply supported girder, and increases the flexural stiffness of the section near cantilever free end.

Key words: II-shaped girder; shear lag; axial displacement; solid element; flexural stiffness

0 引言

II 形梁以其自重轻、桥面宽等特点,在斜拉桥中得到了广泛的应用,但因其宽高比大,剪力滞效应突出,故研究其在压弯作用下剪力滞的计算方法便尤为重要。

针对剪力滞效应,很多学者进行过大量研究工作。在数值方法方面,张元海等^[1-3]在 Reissner^[4]提出的能量变分法基础上,针对有限梁段法做了大量的研究论证与完善改进,但主要集中在箱形截面,对 II 形梁的讨论尚不充分。在解析理论方面,Chen 等^[5]提出了一种导出翼缘变形函数的剪力滞闭合解,无需假设剪滞翘曲位移,但不适于压弯构件。

对于截面纵向位移模式的研究,Lin 等^[6]提出了一种具有高阶多项式剪滞翘曲位移和轴向位移修正的截面纵向位移函数,并引入基于曲率的有效模量^[7]对钢箱梁的弹塑性弯曲变形进行分析;Zhang 等^[8-10]则基于面内剪切变形和截面应力轴向平衡,同样提出了一种考虑轴向位移修正的截面纵向位移函数,但所提出的轴向位移修正项均不独立,无法应用于压弯构件。Zhu 等^[11-12]基于 Dezi 等^[13-14]提出的二次剪滞翘曲位移,对 II 形组合梁也提出了一种考虑轴向位移的剪力滞解析模型,但未能给出其闭合解。

在压弯作用下,程翔云等^[15]通过引入梁柱效应轴向荷载势能和纵向位移差等方法分别考虑压弯和轴压作用下的剪力滞效应;Chang 等^[16-19]对预应力混凝土土箱梁的剪力滞效应进行了研究;李乔等^[20-22]对斜拉桥中 II 形主梁的剪力滞进行了试验分析。压弯作用下 II 形梁剪力滞的研究仍多采用板壳、实体有限元的数值方法或模型试验和现场实测^[21],理论研究尚显不足。

II 形梁剪力滞效应的现有成果往往仅关注弯曲作用下的剪力滞,对有轴向力的主梁(如斜拉桥),分析也局限于对轴力的单独分析,或在能量变分法中简单涉及梁柱效应的轴向荷载势能,并未真正将轴向位移作为独立自由度,忽略了由剪力滞产生的形心轴与中性轴相互分离的现象,从而不能合理分析压弯作用下 II 形梁的剪力滞效应。本文针对剪力

滞产生的形心轴与中性轴相互分离的现象,在截面纵向位移函数中引入轴向位移,以描述弯曲作用下 II 形主梁中性轴的偏移现象;同时解决压弯作用下轴向位移、竖向位移与剪力滞位移间相互耦合的关系,形成一套合理的压弯 II 形梁剪力滞分析方法。

1 基本假定

针对剪力滞翘曲位移函数,文献[8]论证了弯曲作用下面内剪切变形使顶板纵向位移沿其板宽方向按二次抛物线分布,验证了早期 Reissner 所选剪力滞翘曲位移函数的合理性;文献[15]同样也采用二次抛物线作为轴力作用下剪力滞的翘曲位移模式。由于轴力和弯曲均会产生剪力滞效应,本文假设其翘曲位移模式均为二次抛物线,并将两者产生的剪力滞效应合并,用剪力滞位移表征压弯作用下的剪力滞。

建立坐标系 $Oxyz$,如图 1 所示,其中 b 为顶板半宽, t_w 为腹板厚度, t_f 为顶板厚度, h_u 为 y 轴距梁底边的距离, h_b 为 y 轴距梁顶边的距离。

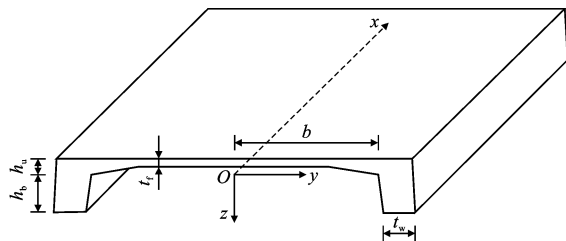


图 1 II 形梁横截面与坐标系

Fig. 1 Cross Section of II-shaped Girder and Coordinate System

为完整描述 II 形梁截面纵向位移,如图 2 所示需要 3 个独立位移变量: $z=0$ 处腹板的轴向位移 $u(x)$ 、截面转角 $\theta(x)$ 和剪切转角最大差值 $\varphi(x)$ (压

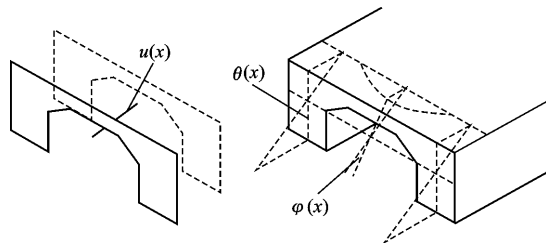


图 2 II 形梁截面独立位移变量

Fig. 2 Independent Displacement Variables on Section of II-shaped Girder

弯作用下的剪力滞位移)。

各位移变量的正向约定为: $u(x)$ 沿 $+x$ 为正; 竖向位移 $v(x)$ 沿 $+z$ 为正; $\theta(x)$ 和 $\varphi(x)$ 绕 $+y$ 旋转为正。 $v(x)$ 和 $\theta(x)$ 有如下关系

$$\theta(x) = -v'(x) \quad (1)$$

只有引入轴向位移 $u(x)$ 才能完整描述 Π 形梁截面纵向位移, 故假设截面纵向位移为

$$\left. \begin{aligned} u_t(x, y, z) &= u(x) + z\theta(x) + z(1 - \frac{y^2}{b^2})\varphi(x) \\ u_w(x, z) &= u(x) + z\theta(x) \end{aligned} \right\} \quad (2)$$

式中: $u_t(x, y, z)$ 为顶板纵向位移函数; $u_w(x, z)$ 为腹板纵向位移函数。

腹板仍满足平截面假定, 对顶板而言, 板内纵向纤维的竖向挤压变形、板平面外的剪切变形及横向弯曲、横向应变均属微量, 忽略不计。

2 控制微分方程的建立

图 3 为主梁承受的外荷载(荷载方向均为正向), 其中 $q_v(x)$ 为竖向分布荷载, $q_u(x)$ 为轴向分布荷载, l 为梁体长度, N_0, Q_0, M_0, S_0 分别为 $x=0$ 处的轴力、剪力、弯矩和剪力滞矩, N_1, Q_1, M_1, S_1 分别为 $x=l$ 处的轴力、剪力、弯矩和剪力滞矩。主梁的荷载势能 V 为

$$V = -(\delta^e)^T f^e - \int_0^l q_v(x)v(x)dx - \int_0^l q_u(x)u(x)dx \quad (3)$$

$$(\delta^e)^T f^e = -N(x)u(x)|_0^l + Q(x)v(x)|_0^l + M(x)\theta(x)|_0^l + S(x)\varphi(x)|_0^l \quad (4)$$

式中: δ^e, f^e 分别为梁端位移及其相应的梁端荷载; $N(x)$ 为轴力; $Q(x)$ 为剪力; $M(x)$ 为弯矩; U_w 为腹板形变势能; U_t 为顶板形变势能; $S(x)$ 为与 $\varphi(x)$ 对应的剪力滞矩。

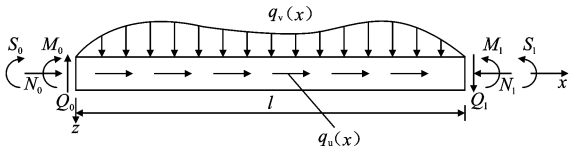


图 3 荷载方向

Fig. 3 Load Direction

主梁的形变势能 U 为

$$U = U_w + U_t \quad (5)$$

$$\left. \begin{aligned} U_w &= \int_{-h_u}^{h_u} \int_0^l t_w E \epsilon_w^2 dx dz \\ U_t &= \int_{-h_u}^{-(h_u-t_t)} \int_0^l \int_0^b (E \epsilon_t^2 + G \gamma_t^2) dx dy dz \end{aligned} \right\} \quad (6)$$

式中: U_w 为腹板形变势能; U_t 为顶板形变势能; E 为弹性模量; G 为剪切模量; ϵ_w 为腹板正应变; ϵ_t, γ_t

分别为顶板的正应变和切应变。

$\epsilon_w, \epsilon_t, \gamma_t$ 具体形式为

$$\left. \begin{aligned} \epsilon_w &= \frac{\partial u_w(x, z)}{\partial x} \\ \epsilon_t &= \frac{\partial u_t(x, y, z)}{\partial x} \\ \gamma_t &= \frac{\partial u_t(x, y, z)}{\partial y} \end{aligned} \right\} \quad (7)$$

由式(3), (5)可以得到主梁的总势能 Π 为

$$\begin{aligned} \Pi = V + U = & -(\delta^e)^T f^e - \int_0^l q_v(x)v(x)dx - \int_0^l q_u(x)u(x)dx + \frac{1}{2} \int_0^l EA(u')^2 dx + \\ & \frac{1}{2} \int_0^l EI(v'')^2 dx + \frac{1}{2} \int_0^l EI_\varphi(\varphi')^2 dx - \\ & \frac{2}{3} \int_0^l EI_t \varphi' v'' dx + \frac{2}{3} \int_0^l ES_t u' \varphi' dx + \\ & \frac{2}{3b^2} \int_0^l GI_t \varphi^2 dx \end{aligned} \quad (8)$$

式中: A 为横截面面积; I 为截面对 y 轴的惯性矩; I_t 为顶板对 y 轴的惯性矩; S_t 为顶板对 y 轴的面积矩; $I_\varphi = 8I_t/15$ 。

对式(8)求变分, 得

$$\left. \begin{aligned} EAu'' + \frac{2}{3}ES_t\varphi'' + q_u(x) &= 0 \\ EIv^{(4)} - \frac{2}{3}EI_t\varphi^{(3)} - q_v(x) &= 0 \\ EI_\varphi\varphi'' - \frac{2}{3}EI_tv'' + \frac{2}{3}ES_tu'' - \frac{4}{3b^2}GI_t\varphi &= 0 \end{aligned} \right\} \quad (9)$$

式(9)即为分析 Π 形梁时位移 $u(x), v(x), \varphi(x)$ 之间相互耦合的剪力滞控制微分方程组。

3 微分方程组的解

本文采用 D 算子方法求解控制微分方程, 将式(9)整理得

$$\left. \begin{aligned} EIv^{(4)} - \frac{2}{3}EI_t\varphi^{(3)} - q_v(x) &= 0 \\ E(\frac{4S_t^2}{9A} - I_\varphi)\varphi'' + \frac{4GI_t}{3b^2}\varphi + \frac{2}{3}EI_tv^{(3)} &= -\frac{2}{3}\frac{S_t}{A}q_u(x) \end{aligned} \right\} \quad (10)$$

引入算子 $D^n = d^n/dx^n$ 将式(10)转化为

$$\left. \begin{aligned} D^4v - \frac{2I_t}{3I}D^3\varphi &= \frac{q_v(x)}{EI} \\ D^3v + [\frac{2G}{Eb^2} - \frac{4}{5}(1 - \frac{5S_t^2}{6I_tA})D^2]\varphi &= -\frac{S_t}{AEI_t}q_u(x) \end{aligned} \right\} \quad (11)$$

令

$$\left. \begin{aligned} z_0 &= \frac{S_f}{A}, m = 1 - \frac{5S_f z_0}{6I_f} \\ n &= 1/(m - \frac{5}{6} \frac{I_f}{I}), k^2 = \frac{5Gn}{2Eb^2} \end{aligned} \right\} \quad (12)$$

当 $q_v(x)$ 为均布荷载, $q_u(x)$ 为线性荷载, 即 $q_v(x) = q_v, q_u(x) = q_0 + q_u x$ 时, 将式(11)转化为

$$\left. \begin{aligned} (D^6 - k^2 D^4)v &= -\frac{k^2}{EI} q_v \\ (D^3 - k^2 D)\varphi &= \frac{5nz_0}{4EI_f} q_u + \frac{5n}{4EI} q_v \end{aligned} \right\} \quad (13)$$

求解式(13), 得剪力滞控制微分方程组的解为

$$\left. \begin{aligned} u(x) &= -c_1 \frac{z_0}{\beta k^3} e^{kx} + c_2 \frac{z_0}{\beta k^3} e^{-kx} - \frac{q_0}{2EA} x^2 - \\ &\quad \frac{q_u}{6EA} x^3 + c_7 + c_8 x \\ v(x) &= \frac{c_1}{k^4} e^{kx} + \frac{c_2}{k^4} e^{-kx} + c_3 + c_4 x + \\ &\quad c_5 x^2 + c_6 x^3 + \frac{q_v}{24EI} x^4 \\ \varphi(x) &= c_1 \frac{3}{2\beta k^3} e^{kx} - c_2 \frac{3}{2\beta k^3} e^{-kx} - \\ &\quad c_6 \frac{3Eb^2}{G} - \frac{b^2 z_0}{2GI_f} q_u(x) - \frac{q_v b^2}{2GI} x \end{aligned} \right\} \quad (14)$$

式中: $c_1 \sim c_8$ 均为积分常数; $\beta = \frac{I_f}{I^0}$ 。

4 边界条件

与式(9)对应的边界条件为

$$\left. \begin{aligned} [N(x) + EAu' + \frac{2}{3} ES_f \varphi'] \delta u \Big|_0^l &= 0 \\ [Q(x) + EIv'' - \frac{2}{3} EI_f \varphi''] \delta v \Big|_0^l &= 0 \\ [M(x) + EIv' - \frac{2}{3} EI_f \varphi'] \delta v' \Big|_0^l &= 0 \\ [S(x) - EI_\varphi \varphi' + \frac{2}{3} EI_f v'' - \frac{2}{3} ES_f u'] \delta \varphi \Big|_0^l &= 0 \end{aligned} \right\} \quad (15)$$

在边界上, 梁端力应该满足以下公式

$$\left. \begin{aligned} N(x) &= -EAu' - \frac{2}{3} ES_f \varphi' \\ Q(x) &= -EIv'' + \frac{2}{3} EI_f \varphi'' \\ M(x) &= -EIv' + \frac{2}{3} EI_f \varphi' \\ S(x) &= EI_\varphi \varphi' - \frac{2}{3} EI_f v'' + \frac{2}{3} ES_f u' \end{aligned} \right\} \quad (16)$$

图 4 为简支梁示意。



图 4 简支梁示意

Fig. 4 Schematic Diagram of Simply Supported Beam

简支梁位移边界条件为

$$v(0) = 0, v(l) = 0, u(0) = 0 \quad (17)$$

简支梁力边界条件为

$$\begin{aligned} M(0) &= 0, M(l) = 0, S(0) = 0 \\ S(l) &= 0, N(l) = N_1 \end{aligned} \quad (18)$$

图 5 为悬臂梁示意。



图 5 悬臂梁示意

Fig. 5 Schematic Diagram of Cantilever Beam

悬臂梁位移边界条件为

$$u(0) = 0, v(0) = 0, \theta(0) = 0, \varphi(0) = 0 \quad (19)$$

悬臂梁力边界条件为

$$N(l) = N_1, Q(l) = 0, M(l) = 0, S(l) = 0 \quad (20)$$

当 $N_1 = 0$ 时, 简支梁和悬臂梁为弯曲状态; 当 $N_1 \neq 0$ 时, 简支梁和悬臂梁为压弯状态。将式(14), (16)代入式(17)~(20), 即可求出积分常数 $c_1 \sim c_8$ 。

5 算例分析

结合图 6 所示的主梁断面尺寸, 分别采用本文方法和 ANSYS 软件提供的 Solid185 实体单元分析简支梁和悬臂梁在图 7 所示荷载下考虑轴向位移时的剪力滞效应。材料的弹性模量 $E = 3.55 \times 10^7$ kPa, 剪切模量 $G = 0.4E$ 。

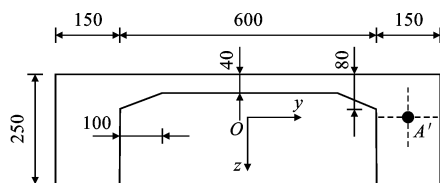


图 6 主梁横断面尺寸 (单位: cm)

Fig. 6 Dimension of Cross Section of Girder (Unit: cm)

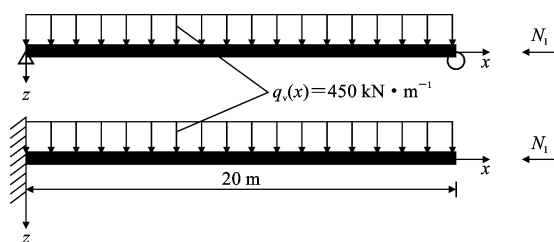


图 7 荷载示意

Fig. 7 Load Diagram

为实现实体单元与本文方法间的比较,选择图 6 中的 A' 点(腹板中面 $z=0$ 处)作为实体单元响应输出位置,以保证这 2 种计算方法之间具有可比性。

对初等梁理论而言,不考虑梁柱效应,将弯曲引起的轴向位移和轴力引起的轴向位移线性叠加,轴向位移为

$$u_e(x) = \frac{1}{2} \int_0^x (v_e')^2 dx + \frac{N_1}{EA} x \tag{21}$$

式中: $u_e(x)$ 为压弯作用下初等梁的轴向位移; $v_e(x)$ 为初等梁的竖向位移。

对于简支梁, $v_e(x)$ 为

$$v_e(x) = \frac{q_v l^3}{24EI} x - \frac{q_v l}{12EI} x^3 + \frac{q_v}{24EI} x^4 \tag{22}$$

对于悬臂梁, $v_e(x)$ 为

$$v_e(x) = \frac{q_v l^2}{4EI} x^2 - \frac{q_v l}{6EI} x^3 + \frac{q_v}{24EI} x^4 \tag{23}$$

在弯曲作用 [$N_1=0, q_v(x)=450 \text{ kN} \cdot \text{m}^{-1}$] 和压弯作用 [$N_1=0, q_v(x)=450 \text{ kN} \cdot \text{m}^{-1}$] 下,分别采用实体单元、本文方法以及初等梁理论对图 7 所示的简支梁和悬臂梁进行轴向位移和竖向位移比较,如图 8,9 所示。

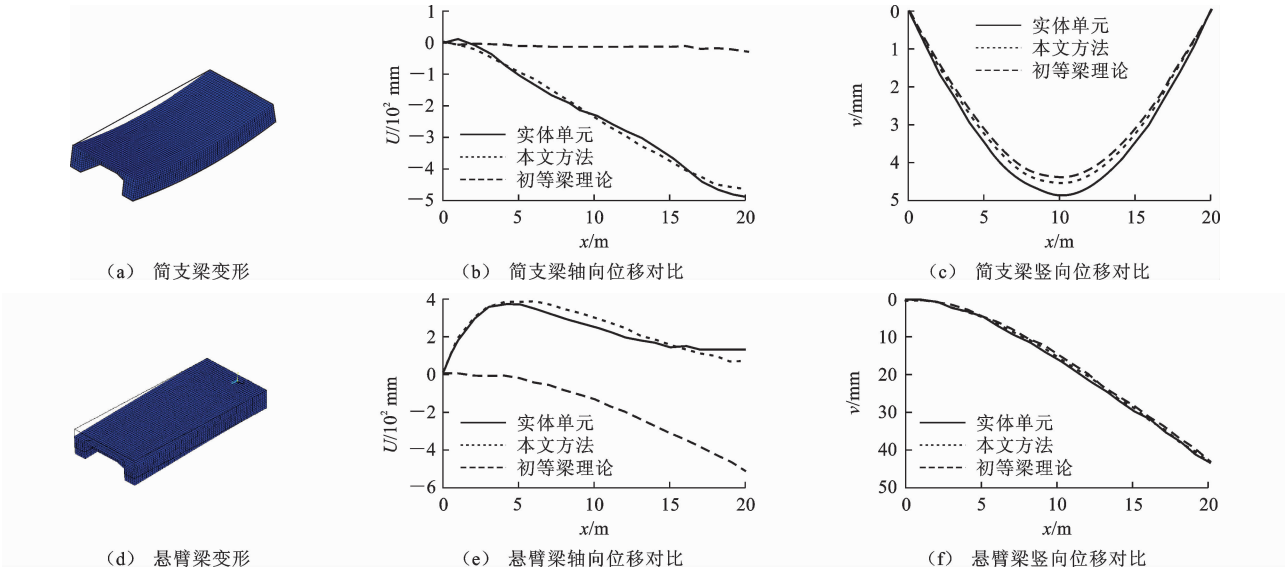


图 8 弯曲作用下实体单元、本文方法和初等梁理论的位移对比

Fig. 8 Displacement Comparison of Solid Element, Present Method and Elementary Beam Theory Under Bending

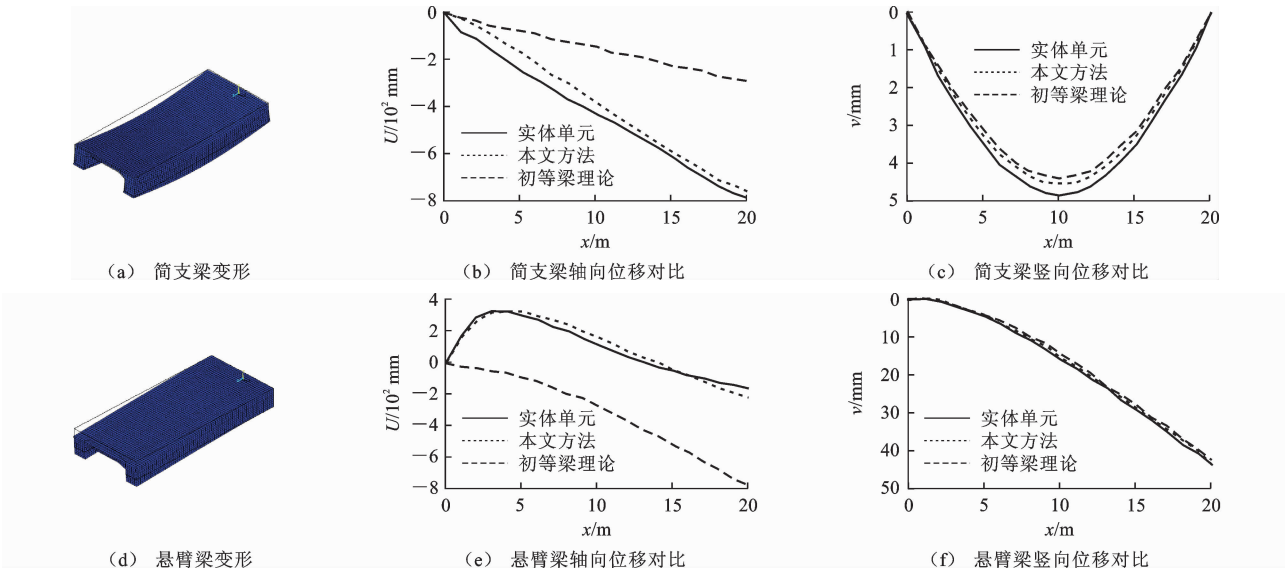


图 9 压弯作用下实体单元、本文方法和初等梁理论的位移对比

Fig. 9 Displacement Comparison of Solid Element, Present Method and Elementary Beam Theory Under Axial Compression and Bending

研究表明,在弯曲作用和压弯作用下,相对于初等梁理论而言,本文方法与实体单元分析结果吻合良好,尤其能够正确反映轴向位移沿梁长的变化规律,验证了本文方法的适用性,能适应弯曲和压弯等不同的受力状态。

当中性轴位于腹板内时,有

$$\epsilon_w = u'(x) + z\theta'(x) = 0 \quad (24)$$

将式(1),(14)代入式(24)即可求得中性轴沿梁长的变化曲线为

$$z = \left[c_8 - \frac{1}{EA} (q_0 x + \frac{q_u}{2} x^2) - \frac{z_0}{\beta k^2} (c_1 e^{kx} + c_2 e^{-kx}) \right] \cdot \left[\frac{1}{k^2} (c_1 e^{kx} + c_2 e^{-kx}) + 2c_5 + 6c_6 x + \frac{q_v x^2}{2EI} \right]^{-1} \quad (25)$$

图10为考虑剪力滞效应时简支梁和悬臂梁中性轴沿梁长的变化曲线。显然,剪力滞使得II形梁中性轴和形心轴相互分离,偏离程度与偏离方向沿梁长逐渐变化。

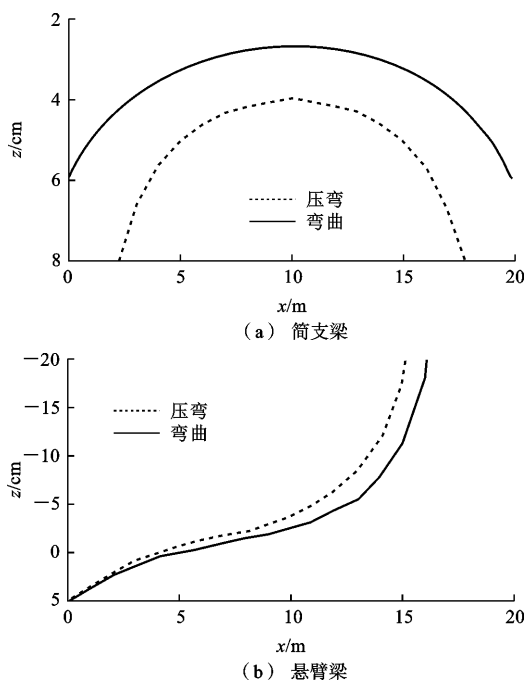


图10 考虑剪力滞影响的中性轴变化曲线

Fig. 10 Variation Curves of Neutral Axis Considering Shear Lag Effect

在弯曲作用下,剪力滞使得简支梁各截面中性轴向下偏移,且梁端下移量大,梁内下移量小,截面弯曲刚度梁端降幅大,梁内降幅小。对悬臂梁而言,剪力滞使固定端一侧1/4跨度内截面中性轴向下偏移,截面弯曲刚度减小;自由端一侧3/4跨度内中性轴向上偏移,截面弯曲刚度增加。

在压弯作用下,轴向压力将使截面中性轴向受拉一侧偏移,但不改变截面弯曲刚度;此外轴向压力

引起的剪力滞也会影响截面中性轴的偏移,并改变截面弯曲刚度。对简支梁而言,压弯作用下截面中性轴整体下移,且梁端下移量大,梁内下移量小,表明梁端附近截面弯曲刚度进一步减小,梁内截面弯曲刚度几乎不变。对悬臂梁而言,压弯作用下截面中性轴上移,固定端一侧上移量很小,忽略不计,自由端一侧上移量较大,表明固定端一侧截面弯曲刚度几乎不变,自由端一侧截面弯曲刚度进一步增加。

6 结 语

(1)本文在截面纵向位移函数中引入轴向位移 $u(x)$,以解决中性轴沿梁长在横截面上下变化,以及与形心轴相互分离的问题,并给出了中性轴沿梁长的变化;同时为压弯作用下轴向位移、竖向位移与剪力滞位移间相互耦合关系的分析奠定了理论基础。

(2)本文方法能够较好地与实体单元分析结果吻合,相对于初等梁理论而言能更好地反映II形梁的位移状态。

(3)剪力滞效应使得II形梁中性轴和形心轴相互分离,截面不再绕形心轴转动,且偏离程度与偏离方向沿梁长变化。

(4)在弯曲作用下,剪力滞使简支梁中性轴向下偏移,截面弯曲刚度减小。对悬臂梁而言,剪力滞使固定端一侧1/4跨度内中性轴下移,截面弯曲刚度减小;自由端一侧3/4跨度内中性轴上移,截面弯曲刚度增加。

(5)在压弯作用下,轴向压力引起的剪力滞使得简支梁梁端附近截面弯曲刚度进一步减小,而梁内截面弯曲刚度几乎不变;对悬臂梁而言,固定端一侧截面弯曲刚度几乎不变,而自由端一侧截面弯曲刚度将进一步增加。

参考文献:

References:

- [1] 张元海,李 乔.考虑剪滞变形时箱形梁广义力矩的数值分析[J].工程力学,2010,27(4):30-36.
ZHANG Yuan-hai, LI Qiao. Numerical Analysis of Generalized Moment in Box Girder Considering Shear Lag Deformation[J]. Engineering Mechanics, 2010, 27 (4): 30-36.
- [2] 张元海,李 琳,林丽霞,等.以附加挠度作为广义位移时薄壁箱梁剪力滞效应的梁段有限元分析[J].土木工程学报,2013,46(10):100-107.
ZHANG Yuan-hai, LI Lin, LIN Li-xia, et al. Beam-

- segment Finite Element Analysis on Shear Lag Effect of Thin-walled Box Girder Adopting Additional Deflection as Generalized Displacement[J]. China Civil Engineering Journal, 2013, 46(10): 100-107.
- [3] LUO Q Z, LI Q S, TANG J. Shear Lag in Box Girder Bridges[J]. Journal of Bridge Engineering, 2002, 7(5): 308-313.
- [4] REISSNER E. Analysis of Shear Lag in Box Beams by the Principle of Minimum Potential Energy[J]. Quarterly of Applied Mathematics, 1946, 4(3): 268-278.
- [5] CHEN J, SHEN S L, YIN Z Y, et al. Closed-form Solution for Shear Lag with Derived Flange Deformation Function[J]. Journal of Constructional Steel Research, 2014, 102: 104-110.
- [6] LIN Z, ZHAO J. Least-work Solutions of Flange Normal Stresses in Thin-walled Flexural Members with High-order Polynomial[J]. Engineering Structures, 2011, 33(10): 2754-2761.
- [7] LIN Z, ZHAO J. Modeling Inelastic Shear Lag in Steel Box Beams[J]. Engineering Structures, 2012, 41: 90-97.
- [8] ZHANG Y H, LIN L X. Shear Lag Analysis of Thin-walled Box Girders Based on a New Generalized Displacement[J]. Engineering Structures, 2014, 61: 73-83.
- [9] ZHANG Y H, LIN L X. Shear Lag Analysis of Thin-walled Box Girders Adopting Additional Deflection as Generalized Displacement[J]. Journal of Engineering Mechanics, 2014, 140(4): 04014005.
- [10] 周茂定, 李丽园, 张元海. 薄壁箱梁的剪力滞翘曲位移函数研究[J]. 中国公路学报, 2015, 28(6): 67-73.
- ZHOU Mao-ding, LI Li-yuan, ZHANG Yuan-hai. Research on Shear-lag Warping Displacement Function of Thin-walled Box Girders[J]. China Journal of Highway and Transport, 2015, 28(6): 67-73.
- [11] ZHU L, NIE J G, LI F X, et al. Simplified Analysis Method Accounting for Shear-lag Effect of Steel-concrete Composite Decks[J]. Journal of Constructional Steel Research, 2015, 115(7): 62-80.
- [12] ZHU L, NIE J G, JI W Y. Positive and Negative Shear Lag Behaviors of Composite Twin-girder Decks with Varying Cross-section[J]. Science China Technological Sciences, 2017, 60(1): 116-132.
- [13] DEZI L, GARA F, LEONI G. Shear-lag Effect in Twin-girder Composite Decks[J]. Steel and Composite Structures, 2003, 3(2): 111-122.
- [14] DEZI L, GARA F, LEONI G. Effective Slab Width in Prestressed Twin-girder Composite Decks[J]. Journal of Structural Engineering, 2006, 132(9): 1358-1370.
- [15] 程翔云, 罗旗帜. 箱梁在压弯荷载共同作用下的剪力滞[J]. 土木工程学报, 1991, 24(1): 52-64.
- CHENG Xiang-yun, LUO Qi-zhi. Shear Lag of Box Girders Under Combined Bending and Axial Loading[J]. China Civil Engineering Journal, 1991, 24(1): 52-64.
- [16] CHANG S T. Shear Lag Effect in Simply Supported Prestressed Concrete Box Girder[J]. Journal of Bridge Engineering, 2004, 9(2): 178-184.
- [17] ZHOU S J. Shear Lag Analysis in Prestressed Concrete Box Girders[J]. Journal of Bridge Engineering, 2011, 16(4): 500-512.
- [18] ZHOU S J. Effect of Construction Method on Shear Lag in Prestressed Concrete Box Girders[J]. Mathematical Problems in Engineering, 2012(4): 1409-1431.
- [19] 蔺鹏臻, 方炜彬, 杨子江, 等. 预应力作用下箱梁桥的剪力滞效应研究[J]. 中国公路学报, 2015, 28(5): 101-107.
- LIN Peng-zhen, FANG Wei-bin, YANG Zi-jiang, et al. Shear Lag Effect Analysis of Box Girder Bridge Under Prestressing Effect[J]. China Journal of Highway and Transport, 2015, 28(5): 101-107.
- [20] 李乔, 唐亮, 黄道全, 等. 斜拉桥 II 形截面 PC 主梁剪力滞模型试验研究[J]. 桥梁建设, 2004, 34(5): 7-10.
- LI Qiao, TANG Liang, HUANG Dao-quan, et al. Model Test and Study of Shear Lag of Prestressed Concrete Main Girder of Cable-stayed Bridge with II-shape Cross Section[J]. Bridge Construction, 2004, 34(5): 7-10.
- [21] 万臻, 李乔, 毛学明. II 型截面主梁斜拉桥剪力滞效应试验研究[J]. 西南交通大学学报, 2004, 39(5): 623-627.
- WAN Zhen, LI Qiao, MAO Xue-ming. Experimental Research of Shear Lag Effect of Cable-stayed Bridge with II Cross-section Main Girder[J]. Journal of Southwest Jiaotong University, 2004, 39(5): 623-627.
- [22] 张永健, 黄平明, 贺志高. II 形梁斜拉桥主梁剪力滞特性试验[J]. 长安大学学报: 自然科学版, 2008, 28(6): 51-54.
- ZHANG Yong-jian, HUANG Ping-ming, HE Zhigao. Test on Shear Lag of Cable-stayed Bridge with II-shaped Cross Section Main Girder[J]. Journal of Chang'an University: Natural Science Edition, 2008, 28(6): 51-54.