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厚圆柱扁壳位移型基本方程及控制方程分析

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摘要:基于考虑横向剪切变形的厚壳位移型基本方程及扁壳基本假定,建立了以 5 个中面位移为 5 个独立变量的厚扁壳及厚圆柱扁壳位移型基本方程。对厚圆柱扁壳进行了动力分析,首先引入 3 个辅助位移函数,同时运用柯西-黎曼条件,将 5 个二阶微分方程变形为 1 个二阶和 2 个四阶过渡微分方程;然后引入另一辅助位移函数,建立其解耦的控制微分方程;最后利用 4 个辅助位移函数求出 5 个位移分量。结果表明,厚扁壳的位移型基本方程退化为厚圆柱扁壳及薄圆柱扁壳的位移型方程是正确的,且所推导的方程具有通用性。

关键词:横向剪切变形;厚壳;厚圆柱扁壳;位移函数;位移型基本方程;控制方程

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Displacement Fundamental Equations and Analysis of Governing Equations of Thick Cylindrical Shallow Shells

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Abstract: The displacement fundamental equations of the thick shallow shells and thick cylindrical shallow shells concerning five independent variables, ie five middle surface displacements were established based on the displacement fundamental equations of the thick shells by transverse shearing deformation and basic hypothesis on shallow shells. Authors analyzed thick cylindrical shallow shells' dynamic characteristics by introducing three assistant displacement functions, ie authors converted five second-order differential equations into a second-order differential equation and two fourth-order transition differential equations using Cauchy-Riemann condition, and then introduced another assistant displacement functions to build its decoupled governing differential equations and obtained five displacement components through four assistant displacement functions. The displacements of equations of thick shallow shells degenerate to the displacement equations of the thick cylindrical shallow shells and thin cylindrical shallow shells, which are proved to be right and demonstrate the generality of derived equations.

Key words: transverse shearing deformation; thick shell; thick cylindrical shallow shell; displacement function; displacement fundamental equation; governing equation

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0 引 言

在工程实践中,复合材料在壳体中的应用为板壳结构的计算提出了新的理论问题[1-4],由于这些复合材料一般都具有较低的横向剪切刚度,以前在板壳结构计算中所使用的横向刚度假定[5-11]的经典理论将会产生很大的误差,因此需要考虑横向剪切变形的影响;再者,由于实际工程中板壳结构壁厚的增加往往超出薄壁的应用范围,因此也需要考虑横向剪切变形的影响。鉴于此,考虑横向剪切变形的影响是十分重要的。

用弹性壳体的位移型基本方程求解壳体问题非常方便,随着壳体应用范围的扩展,越来越显示出其位移型基本方程的重要性。关于考虑横向剪切变形的厚壳,各国已有研究,但仅局限于厚圆柱壳的位移型方程的建立,笔者在文献[12]和文献[13]中虽然给出了正交曲线坐标下的一般厚壳、厚圆柱壳、厚圆锥壳及厚球壳的位移型基本方程,但有关厚壳的位移解分析目前还是空白。

笔者从厚壳位移型基本方程出发,引入扁壳基本假定,建立了以 3 个中面位移 u_1、u_2、w 及 2 个中面转角 φ_1、φ_2 为 5 个独立变量的中厚扁壳的位移型基本方程,并建立了厚圆柱扁壳的位移型基本方程。为了求解厚圆柱扁壳的位移型基本方程,引入了 4 个辅助位移函数 $F(x,y,t)$、$f(x,y,t)$、$F_1(x,y,t)$、$F_2(x,y,t)$,建立其控制方程,最后求出 5 个位移分量 $u_1(x,y,t)$、$u_2(x,y,t)$、$w(x,y,t)$、$\varphi_1(x,y,t)$、$\varphi_2(x,y,t)$。

1 基本微分方程

在文献[12]的厚壳位移型基本方程中引入扁壳基本假定,则有厚度为 h、荷载为 q_1、q_2、q_n 的厚扁壳的位移型基本方程

$$\frac{1}{A_1}\frac{\partial}{\partial\alpha_1}\{\frac{1}{A_1A_2}[\frac{\partial}{\partial\alpha_1}(A_2u_1)+\frac{\partial}{\partial\alpha_2}(A_1u_2)]\}+\frac{1}{A_1}\cdot \frac{\partial}{\partial\alpha_1}[(\frac{1}{R_1}+\frac{1}{R_2})w]+\frac{1-\mu}{2}\frac{1}{A_2}\frac{\partial}{\partial\alpha_2}\{\frac{1}{A_1A_2}\cdot [\frac{\partial}{\partial\alpha_2}(A_1u_1)-\frac{\partial}{\partial\alpha_1}(A_2u_2)]\}-(1-\mu)\cdot \frac{1}{A_1}\frac{1}{R_2}\frac{\partial W}{\partial\alpha_1}+(1-\mu)\frac{u_1}{R_1R_2}+\frac{1}{K}q_1=\frac{1}{K}\rho J_0\ddot{u}_1 \quad (1)$$

$$\frac{1}{A_2}\frac{\partial}{\partial\alpha_2}\{\frac{1}{A_1A_2}[\frac{\partial}{\partial\alpha_1}(A_2u_1)+\frac{\partial}{\partial\alpha_2}(A_1u_2)]\}+\frac{1}{A_2}\cdot \frac{\partial}{\partial\alpha_2}[(\frac{1}{R_1}+\frac{1}{R_2})w]+\frac{1-\mu}{2}\frac{1}{A_1}\frac{\partial}{\partial\alpha_1}\{\frac{1}{A_1A_2}\cdot [\frac{\partial}{\partial\alpha_1}(A_2u_2)-\frac{\partial}{\partial\alpha_2}(A_1u_1)]\}-(1-\mu)\cdot \frac{1}{A_2}\frac{1}{R_1}\frac{\partial W}{\partial\alpha_2}+(1-\mu)\frac{u_2}{R_1R_2}+\frac{1}{K}q_2=\frac{1}{K}\rho J_0\ddot{u}_2 \quad (2)$$

$$(\frac{1}{R_1}+\frac{1}{R_2})\frac{1}{A_1A_2}[\frac{\partial}{\partial\alpha_1}(A_2u_1)+\frac{\partial}{\partial\alpha_2}(A_1u_2)]- (1-\mu)\frac{1}{A_1A_2}[\frac{\partial}{\partial\alpha_1}(\frac{1}{R_2}A_2u_1)+\frac{\partial}{\partial\alpha_2}(\frac{1}{R_1}\cdot A_1u_2)]+(\frac{1}{R_1^2}+\frac{2\mu}{R_1R_2}+\frac{1}{R_2^2})w-\frac{1}{K}\cdot \frac{1}{A_1A_2}\frac{\partial}{\partial\alpha_1}[\frac{A_2Gh}{k_\tau}(\varphi_1+\frac{1}{A_1}\frac{\partial w}{\partial\alpha_1})]- \frac{1}{K}\frac{1}{A_1A_2}\frac{\partial}{\partial\alpha_2}[\frac{A_1Gh}{k_\tau}(\varphi_2+\frac{1}{A_2}\frac{\partial w}{\partial\alpha_2})]- \frac{1}{K}q_n=\frac{1}{K}\rho J_0\ddot{w} \quad (3)$$

$$\frac{1}{A_2}\frac{\partial}{\partial\alpha_2}\{\frac{1}{A_1A_2}[\frac{\partial}{\partial\alpha_1}(A_2\varphi_1)+\frac{\partial}{\partial\alpha_2}(A_1\varphi_2)]\}+ \frac{1-\mu}{2}\frac{1}{A_1}\frac{\partial}{\partial\alpha_1}\{\frac{1}{A_1A_2}[\frac{\partial}{\partial\alpha_1}(A_2\varphi_2)- \frac{\partial}{\partial\alpha_2}(A_1\varphi_1)]\}+(1-\mu)\frac{\varphi_2}{R_1R_2}- \frac{1}{D}[\frac{Gh}{k_\tau}(\varphi_2+\frac{1}{A_2}\frac{\partial w}{\partial\alpha_2})]=\frac{1}{D}\rho J_2\ddot{\varphi}_2 \quad (4)$$

$$\frac{1}{A_1}\frac{\partial}{\partial\alpha_1}\{\frac{1}{A_1A_2}[\frac{\partial}{\partial\alpha_1}(A_2\varphi_1)+\frac{\partial}{\partial\alpha_2}(A_1\varphi_2)]\}+ \frac{1-\mu}{2}\frac{1}{A_2}\frac{\partial}{\partial\alpha_2}\{\frac{1}{A_1A_2}[\frac{\partial}{\partial\alpha_2}(A_1\varphi_1)- \frac{\partial}{\partial\alpha_1}(A_2\varphi_2)]\}+(1-\mu)\frac{\varphi_2}{R_1R_2}- \frac{1}{D}[\frac{Gh}{k_\tau}(\varphi_1+\frac{1}{A_1}\frac{\partial w}{\partial\alpha_1})]=\frac{1}{D}\rho J_2\ddot{\varphi}_1 \quad (5)$$

式中:α_1、α_2 为中面正交曲线坐标系的坐标;u_1、u_2 为中面上 $P(\alpha_1,\alpha_2)$ 的位移分量;w 为中面上 $P(\alpha_1,\alpha_2)$ 法线方向 z 的位移分量;φ_1、φ_2 为中面上 $P(\alpha_1,\alpha_2)$ 独立转角;A_1、A_2 及 R_1、R_2 分别为拉梅系数和主曲率半径;$J_0=h$;$J_2=\frac{h^3}{12}$;$K=\frac{Eh}{1-\mu^2}$;$D=\frac{Eh^3}{12(1-\mu^2)}$。此时,横向力就变为 $T_{1n}=\frac{Gh}{k_\tau}(\varphi_1+\frac{\partial w}{\partial x})$ 和 $T_{2n}=\frac{Gh}{k_\tau}(\varphi_2+\frac{\partial w}{R\partial\theta})$。

取圆柱壳坐标系为

$$\alpha_1=x,\alpha_2=\theta$$

则拉梅系数为

$$A_1=1,A_2=R$$

主曲率半径为

$$R_1=\infty,R_2=R$$

$R_1 \mathrm{d}\varphi = \mathrm{d}x, \varphi = \frac{\pi}{2}, \sin\varphi = 1, \cos\varphi = 0$

将上述各变量代入式(1)～(5)，并令 $q_1 = q_2 = 0$，则厚圆柱扁壳的位移型基本方程为

$$\frac{\partial^2 u_1}{\partial x^2} + \frac{1-\mu}{2}\frac{\partial^2 u_1}{R^2\partial\theta^2} + \frac{1+\mu}{2}\frac{\partial^2 u_2}{R\partial x\partial\theta} + \frac{\mu}{R}\frac{\partial w}{\partial x} = \frac{1}{K}\rho J_0 \ddot{u}_1 \tag{6}$$

$$\frac{1+\mu}{2}\frac{\partial^2 u_1}{R\partial x\partial\theta} + \frac{1-\mu}{2}\frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_2}{R^2\partial\theta^2} + \frac{1}{R}\frac{\partial w}{R\partial\theta} = \frac{1}{K}\rho J_0 \ddot{u}_2 \tag{7}$$

$$\frac{\mu}{R}\frac{\partial u_1}{\partial x} + \frac{\partial u_2}{R^2\partial\theta} + \frac{w}{R^2} - \frac{1}{K}\frac{Gh}{k_\tau}\left(\frac{\partial\varphi_1}{\partial x} + \frac{\partial\varphi_2}{R\partial\theta} + \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{R^2\partial\theta^2}\right) - \frac{1}{K}q_n = \frac{1}{K}\rho J_0 \ddot{w} \tag{8}$$

$$\frac{1+\mu}{2}\frac{\partial^2 \varphi_1}{R\partial x\partial\theta} + \frac{1-\mu}{2}\frac{\partial^2 \varphi_2}{\partial x^2} + \frac{\partial^2 \varphi_2}{R^2\partial\theta^2} - \frac{1}{D}\frac{Gh}{k_\tau}\left(\varphi_2 + \frac{\partial w}{R\partial\theta}\right) = \frac{1}{D}\rho J_2 \ddot{\varphi}_2 \tag{9}$$

$$\frac{\partial^2 \varphi_1}{\partial x^2} + \frac{1-\mu}{2}\frac{\partial^2 \varphi_1}{R^2\partial\theta^2} + \frac{1+\mu}{2}\frac{\partial^2 \varphi_2}{R\partial x\partial\theta} - \frac{1}{D}\frac{Gh}{k_\tau}\left(\varphi_1 + \frac{\partial w}{\partial x}\right) = \frac{1}{D}\rho J_2 \ddot{\varphi}_1 \tag{10}$$

将式(9)、(10)剪力项 $\frac{Gh}{k_\tau}\left(\varphi_2 + \frac{\partial w}{R\partial\theta}\right)$、$\frac{Gh}{k_\tau}\left(\varphi_1 + \frac{\partial w}{\partial x}\right)$ 代入式(6)～(8)中，可以得到相应的薄圆柱扁壳的位移型方程，该方程与文献[5]和文献[6]中对应的薄圆柱扁壳方程相同。厚圆柱扁壳的位移型基本方程式(6)～(10)也可以由文献[9]中扁壳的基本假定获得。

2　动力分析

令 $\mathrm{d}s_1 = A_1 \mathrm{d}\alpha_1 = \mathrm{d}x, \mathrm{d}s_2 = A_2 \mathrm{d}\alpha_2 = \mathrm{d}y$，同时取柱壳的半径为 a，由式(6)～(10)得出中厚圆柱扁壳的位移型基本方程的另一种形式为

$$\frac{\partial^2 u_1}{\partial x^2} + \frac{1-\mu}{2}\frac{\partial^2 u_1}{\partial y^2} + \frac{1+\mu}{2}\frac{\partial^2 u_2}{\partial x\partial y} + \frac{\mu}{a}\frac{\partial w}{\partial x} - \frac{1}{K}\rho J_0 \ddot{u}_1 = 0 \tag{11}$$

$$\frac{\partial^2 u_2}{\partial y^2} + \frac{1-\mu}{2}\frac{\partial^2 u_2}{\partial x^2} + \frac{1+\mu}{2}\frac{\partial u_1}{\partial x\partial y} + \frac{1}{a}\frac{\partial w}{\partial y} - \frac{1}{K}\rho J_0 \ddot{u}_2 = 0 \tag{12}$$

$$\frac{1}{a}\left(\frac{\partial u_2}{\partial y} + \frac{w}{a} + \mu\frac{\partial u_1}{\partial x}\right) - \frac{1}{K}\frac{Gh}{k_\tau}\left(\nabla^2 w + \frac{\partial\varphi_1}{\partial x} + \frac{\partial\varphi_2}{\partial y}\right) - \frac{1}{K}q_n = \frac{1}{K}\rho J_0 \ddot{w} \tag{13}$$

$$\frac{\partial^2 \varphi_1}{\partial x^2} + \frac{1-\mu}{2}\frac{\partial^2 \varphi_1}{\partial y^2} + \frac{1+\mu}{2}\frac{\partial^2 \varphi_2}{\partial x\partial y} - \frac{Gh}{k_\tau D}\left(\varphi_1 + \frac{\partial w}{\partial x}\right) = \frac{1}{D}\rho J_2 \ddot{\varphi}_1 \tag{14}$$

$$\frac{\partial^2 \varphi_2}{\partial y^2} + \frac{1-\mu}{2}\frac{\partial^2 \varphi_2}{\partial x^2} + \frac{1+\mu}{2}\frac{\partial^2 \varphi_1}{\partial x\partial y} - \frac{Gh}{k_\tau D}\left(\varphi_2 + \frac{\partial w}{\partial y}\right) = \frac{1}{D}\rho J_2 \ddot{\varphi}_2 \tag{15}$$

令 $\gamma^2 = \frac{1-\mu^2}{E}\rho, k = \frac{1}{k_\tau}\frac{1-\mu}{2}$，于是有 $\frac{Gh}{k_\tau D} = k\frac{12}{h^2}$，$\frac{Gh}{k_\tau K} = k$，则式(11)～(15)可简记为

$$\frac{\partial^2 u_1}{\partial x^2} + \frac{1-\mu}{2}\frac{\partial^2 u_1}{\partial y^2} + \frac{1+\mu}{2}\frac{\partial^2 u_2}{\partial x\partial y} + \frac{\mu}{a}\frac{\partial w}{\partial x} - \gamma^2 \ddot{u}_1 = 0 \tag{16}$$

$$\frac{\partial^2 u_2}{\partial y^2} + \frac{1-\mu}{2}\frac{\partial^2 u_2}{\partial x^2} + \frac{1+\mu}{2}\frac{\partial^2 u_1}{\partial x\partial y} + \frac{1}{a}\frac{\partial w}{\partial y} - \gamma^2 \ddot{u}_2 = 0 \tag{17}$$

$$\frac{1}{a}\left(\frac{\partial u_2}{\partial y} + \frac{w}{a} + \mu\frac{\partial u_1}{\partial x}\right) - k\left(\nabla^2 w + \frac{\partial\varphi_1}{\partial x} + \frac{\partial\varphi_2}{\partial y}\right) - \frac{q_n}{K} = \gamma^2 \ddot{w} \tag{18}$$

$$\frac{\partial^2 \varphi_1}{\partial x^2} + \frac{1-\mu}{2}\frac{\partial^2 \varphi_1}{\partial y^2} + \frac{1+\mu}{2}\frac{\partial^2 \varphi_2}{\partial x\partial y} - k\frac{12}{h^2}\left(\varphi_1 + \frac{\partial w}{\partial x}\right) = \gamma^2 \ddot{\varphi}_1 \tag{19}$$

$$\frac{\partial^2 \varphi_2}{\partial y^2} + \frac{1-\mu}{2}\frac{\partial^2 \varphi_2}{\partial x^2} + \frac{1+\mu}{2}\frac{\partial^2 \varphi_1}{\partial x\partial y} - k\frac{12}{h^2}\left(\varphi_2 + \frac{\partial w}{\partial y}\right) = \gamma^2 \ddot{\varphi}_2 \tag{20}$$

式(16)～(20)即为中厚圆柱扁壳的位移型动力方程，它是十阶微分方程组，求解十分困难，必须做适当的变换将方程进行化简。为此，引入位移函数 $F_1(x,y,t)$、$F_2(x,y,t)$ 和 $f(x,y,t)$，并令

$$u_1 = -\frac{1-\mu}{2a}\left(\mu\frac{\partial^3}{\partial x^3} - \frac{\partial^3}{\partial x\partial y^2} - \frac{2\mu}{1-\mu}\gamma^2\frac{\partial^3}{\partial x\partial t^2}\right)F_1 \tag{21}$$

$$u_2 = -\frac{1-\mu}{2a}\left[(2+\mu)\frac{\partial^3}{\partial x^2\partial y} - \frac{\partial^3}{\partial y^3} - \frac{2}{1-\mu}\gamma^2\frac{\partial^3}{\partial y\partial t^2}\right]F_1 \tag{22}$$

$$w = \left(-\frac{1-\mu}{2}\nabla^2\nabla^2 - \gamma^2\frac{3-\mu}{2}\frac{\partial^2}{\partial t^2}\cdot\nabla^2 + \gamma^4\frac{\partial^4}{\partial t^4}\right)F_1 \tag{23}$$

式中：$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$。将式(21)～(23)代入式(17)、(18)中，发现这 2 个方程自动满足，同时再令

$$\varphi_1=\frac{\partial F_2}{\partial x}-\frac{\partial f}{\partial y},\varphi_2=\frac{\partial F_2}{\partial y}+\frac{\partial f}{\partial x} \tag{24}$$

则由式(19)、(20)可得

$$\frac{\partial}{\partial x}(\nabla^2 F_2-\frac{12k}{h^2}F_2-\frac{12k}{h^2}\gamma^2\ddot{F}_2-\frac{12k}{h^2}w)-\frac{\partial}{\partial y}(\frac{1-\mu}{2}\nabla^2 f-\frac{12k}{h^2}f-\gamma^2\ddot{f})=0 \tag{25}$$

$$\frac{\partial}{\partial y}(\nabla^2 F_2-\frac{12k}{h^2}F_2-\frac{12k}{h^2}\gamma^2\ddot{F}_2-\frac{12k}{h^2}w)+\frac{\partial}{\partial x}(\frac{1-\mu}{2}\nabla^2 f-\frac{12k}{h^2}f-\gamma^2\ddot{f})=0 \tag{26}$$

根据柯西-黎曼条件，由式(25)、(26)分别得出

$$\nabla^2 F_2-\frac{12k}{h^2}F_2-\frac{12k}{h^2}\gamma^2\ddot{F}_2-\frac{12k}{h^2}w=0 \tag{27}$$

$$\frac{1-\mu}{2}\nabla^2 f-\frac{12k}{h^2}f-\gamma^2\ddot{f}=0 \tag{28}$$

将式(23)代入式(27)、(28)中可以建立起函数 $F_1(x,y,t)$ 和 $F_2(x,y,t)$ 应满足的方程，即

$$\nabla^2 F_2-k_0F_2-k_0\gamma^2\ddot{F}_2=k_0(\frac{1-\mu}{2}\nabla^4-\frac{3-\mu}{2}\gamma^2\frac{\partial^2}{\partial t^2}\nabla^2+\gamma^4\frac{\partial^4}{\partial t^4})F_1 \tag{29}$$

式中：$k_0=12k/h^2$。至此，在式(21)～(23)、(27)、(28)的基础上，式(16)、(17)、(19)、(20)均已满足。再将式(21)～(24)代入式(18)中，可得

$$\frac{1}{a}[-\frac{1-\mu}{2a}(1-\mu^2)\frac{\partial^4 F_1}{\partial x^4}-\frac{1-\mu^2}{a}\gamma^2\frac{\partial^4 F_1}{\partial x^2\partial t^2}-\frac{1-\mu}{2a}\gamma^2\frac{\partial^2}{\partial t^2}\nabla^2F_1+\frac{1}{a}\gamma^4\frac{\partial^4 F_1}{\partial t^4}-a\frac{1-\mu}{2}\cdot\gamma^2\frac{\partial^2}{\partial t^2}\nabla^4F_1+a\frac{3-\mu}{2}\gamma^4\frac{\partial^4}{\partial t^4}\nabla^4F_1-a\gamma^6\frac{\partial^6 F_1}{\partial t^6}]-k(\frac{1-\mu}{2}\nabla^6F_1-\frac{3-\mu}{2}\gamma^2\cdot\frac{\partial^2}{\partial t^2}\nabla^4F_1+\gamma^4\frac{\partial^4}{\partial t^4}\nabla^2F_1+\nabla^2F_2)=\frac{1}{K}q_n \tag{30}$$

式(28)～(30)为与式(16)～(20)等效的方程组。为了将它们变为单一的方程，令

$$\left.\begin{aligned}F_1&=(\nabla^2-k_0-k_0\gamma^2\frac{\partial^2}{\partial t^2})F\\F_2&=k_0(\frac{1-\mu}{2}\nabla^4-\frac{3-\mu}{2}\gamma^2\frac{\partial^2}{\partial t^2}\nabla^2+\gamma^4\frac{\partial^4}{\partial t^4})F\end{aligned}\right\} \tag{31}$$

式中：$F(x,y,t)$ 为另一位移函数。将式(31)代入式(30)中，可得函数 $F(x,y,t)$ 应满足的方程，即

$$k\frac{1-\mu}{2}\nabla^8F+T_2(t)\nabla^6F+T_4(t)\nabla^4F+[T_6(t)-\frac{(1-\mu)(1-\mu^2)}{2a^2}\frac{\partial^4}{\partial x^4}]\nabla^2F+T_8(t)F+\frac{(1-\mu)(1-\mu^2)}{2a^2}k_0T_{20}(t)\cdot\frac{\partial^4 F}{\partial x^4}+\frac{1-\mu^2}{a^2}k_0T_{40}(t)\frac{\partial^2 F}{\partial x^2}=-\frac{1}{K}q_n \tag{32}$$

式中

$T_2(t)=(\frac{1-\mu}{2a}-kk_0\frac{1-\mu}{2}-\frac{3-\mu}{2}k)\gamma^2\frac{\partial^2}{\partial t^2}$;

$T_4(t)=(\frac{1-\mu}{2a^2}-\frac{1-\mu}{2a}k_0)\gamma^2\frac{\partial^2}{\partial t^2}+(\frac{3-\mu}{2}kk_0+k-\frac{1-\mu}{2a}k_0-\frac{3-\mu}{2a})\gamma^4\frac{\partial^4}{\partial t^4}$;

$T_6(t)=-(\frac{1-\mu}{2a^2}k_0-\frac{1-\mu^2}{a^2}\frac{\partial^2}{\partial x^2})\gamma^2\frac{\partial^2}{\partial t^2}+(\frac{3-\mu}{2a}k_0-\frac{1-\mu}{2a^2}k_0-\frac{1}{a^2})\gamma^4\frac{\partial^4}{\partial t^4}+(\frac{3-\mu}{2a}k_0+\frac{1}{a}-kk_0)\gamma^6\frac{\partial^6}{\partial t^6}$;

$T_8(t)=\frac{1}{a^2}k_0\gamma^4\frac{\partial^4}{\partial t^4}+(\frac{1}{a^2}k_0-\frac{1}{a}k_0)\gamma^6\frac{\partial^6}{\partial t^6}-\frac{1}{a}\cdot k_0\gamma^8\frac{\partial^8}{\partial t^8}$;

$T_{20}(t)=-1-\gamma^2\frac{\partial^2}{\partial t^2}$;

$T_{40}(t)=-\gamma^2\frac{\partial^2}{\partial t^2}-\gamma^4\frac{\partial^4}{\partial t^4}$。

式(28)、(32)为中厚圆柱扁壳的控制微分方程组，它们的求解是比较简单的。实际上式(28)是一个剪切型方程，而式(32)是一个弯曲型方程。

由式(28)、(32)可以容易得到中厚圆柱扁壳的静力方程，这只要在其中去掉带有 γ 的项就可以直接得到，即

$$\frac{1-\mu}{2}\nabla^2f-k_0f=0 \tag{33}$$

$$k\frac{1-\mu}{2}\nabla^8F-\frac{(1-\mu)(1-\mu^2)}{2a^2}\frac{\partial^4}{\partial x^4}\nabla^2F+\frac{(1-\mu)(1-\mu^2)}{2a^2}k_0\frac{\partial^4 F}{\partial x^4}=-\frac{q_n}{K} \tag{34}$$

当由式(33)、(34)解出 $F(x,y,t)$ 和 $f(x,y,t)$ 之后，由式(31)直接求导得出 $F_1(x,y,t)$ 和 $F_2(x,y,t)$，然后再由式(21)～(24)通过求导得出位移分量 $u_1(x,y,t)$、$u_2(x,y,t)$、$w(x,y,t)$、$\varphi_1(x,y,t)$ 和 $\varphi_2(x,y,t)$。

3 结 语

(1)将厚圆柱扁壳的位移型基本方程式(9)、(10)中的剪力项 $\frac{Gh}{k_\tau}(\varphi_2+\frac{\partial w}{R\partial\theta})$、$\frac{Gh}{k_\tau}(\varphi_1+\frac{\partial w}{\partial x})$ 代入式(6)～(8)中，可以得到相应的薄圆柱扁壳的位移型基本方程，该方程与文献[5]和文献[6]中对应的薄圆柱扁壳方程相同，说明了式(6)～(10)的正确性。

(2)为使厚圆柱扁壳的十阶微分方程式(6)~(10)解耦,首先,引入 4 个辅助位移函数 $F(x,y,t)$、$f(x,y,t)$、$F_1(x,y,t)$、$F_2(x,y,t)$,建立了关于辅助位移函数的控制方程;然后,通过辅助位移函数求解出 5 个位移分量 $u_1(x,y,t)$、$u_2(x,y,t)$、$w(x,y,t)$、$\varphi_1(x,y,t)$和 $\varphi_2(x,y,t)$,这 5 个位移分量的解具有通用性。

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